## NOTES

### Electronic Calculators

1. The use of silent electronic calculators is **expected** in Advanced level Mathematics and Further Mathematics (9233 and 9234) and in AO level Mathematics (8174).

2. Graphic calculators will be permitted only in Advanced level Further Mathematics (9234).

3. More detailed regulations concerning the use of electronic calculators will be issued by the Examinations Branch of the Ministry of Education.

### Lists of Formulae, etc.

Formulae for AO level Mathematics are printed on the question papers.

Candidates entered for A level Mathematics or Further Mathematics will be provided in the examination with a list of formulae.

### Mathematical Instruments

Apart from the usual mathematical instruments, candidates may use flexicurves in all the examinations. In addition, candidates taking Advanced level subjects may use stencils for drawing conic sections, provided that the stencils do not bear formulae not included in the list of formulae provided for use in the examination.

### Mathematical Notation

Attention is drawn to the list of mathematical notation on pages 14-17.

# MATHEMATICS SYLLABUS (9233)

## GCE ADVANCED LEVEL

### Syllabus Aims

The syllabus is intended to provide a framework for an A level course that will enable students to:

1. develop further their understanding of mathematics and mathematical processes in a way that encourages confidence and enjoyment;
2. develop a positive attitude to learning and applying mathematics;
3. acquire and become familiar with appropriate mathematical skills and techniques;
4. appreciate mathematics as a logical and coherent subject with rich interconnections;
5. integrate information technology to enhance the mathematical experience;
6. develop their ability to think clearly, work carefully and communicate mathematical ideas successfully;
7. develop their ability to formulate problems mathematically, interpret a mathematical solution in the context of the original problem and understand the limitations of mathematical models,
8. appreciate how mathematical ideas can be applied in the everyday world;
9. acquire a suitable foundation for further study of mathematics and related disciplines.

### Assessment Objectives

The assessment will test candidates' abilities to:

1. recall, select and use their knowledge of appropriate mathematical facts, concepts and techniques in a variety of contexts;
2. construct rigorous mathematical arguments through appropriate use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions;
3. evaluate mathematical models, including an appreciation of the assumptions made, and interpret, justify and present the results from a mathematical analysis in a form relevant to the original problem.

In addition to the above Objectives, the assessment of the Particle Mechanics topics (in Paper 2 Section B or C) will test candidates' abilities to:

- select the appropriate mechanical principles to apply in a given situation
- understand the assumptions or simplifications which have to be made in order to apply the mechanical principles and comment upon them (especially with reference to modelling a body as a particle)
- use appropriate units throughout

and the assessment of the Probability & Statistics topics (Paper 2 Section B or D) will test candidates' abilities to:

- select an appropriate statistical technique to apply in a given situation
- comment on and interpret statistical results.

It should be noted that individual questions may involve ideas from more than one section of the syllabus, and that topics may be tested in the context of solving problems and in the application of Mathematics.

**9233 MATHEMATICS A LEVEL**

## Scheme of Papers

The examination in Mathematics will consist of two three-hour papers, each carrying 50% of the total mark, and each marked out of 100, as follows:

**Paper 1** A paper consisting of about 14 questions of different marks and lengths, based on the Pure Mathematics syllabus (Sections 1 to 16). Candidates will be expected to answer **all** questions, except that the last question will have a choice of two alternatives (12 marks for each alternative), of which candidates will be expected to answer one of the alternatives.

**Paper 2** A paper consisting of 4 sections, Sections A, B, C and D.

Section A (Pure Mathematics – 34 marks) will consist of about 5 questions of different marks and lengths based on Sections 1 to 16 of the Pure Mathematics syllabus.

Section B (Applied Mathematics – 66 marks) will consist of

- about 4 Mechanics questions of different marks and lengths for 27 marks, based on Sections 1 to 4 of the Particle Mechanics syllabus
- about 4 Statistics questions of different marks and lengths for 27 marks, based on Sections 1 to 4 of the Probability and Statistics syllabus
- a final Either/Or question with two alternatives (12 marks for each alternative), one on Particle Mechanics and one on Probability and Statistics, based respectively on the sections in the preceding two bullets.

Section C (Particle Mechanics – 66 marks) will consist of 6 or 7 questions of different marks and lengths for 54 marks, and a final Either/Or question with two alternatives (12 marks for each alternative). Questions will be based on Sections 1 to 8 of the Particle Mechanics syllabus.

Section D (Probability and Statistics – 66 marks) will consist of 6 or 7 questions of different marks and lengths for 54 marks, and a final Either/Or question question with two alternatives (12 marks for each alternative). Questions will be based on Sections 1 to 7 of the Probability and Statistics syllabus.

For Paper 2, candidates will answer **all** the questions in Section A.
Candidates will also choose **one** of Sections B, C or D, and answer **all** of the questions in that section, except that for the last question in the section chosen, candidates will answer only **one** of the alternatives.

## Special Paper

A Special Paper (3 hours) is available in November. It will consist of two sections, Section A and Section B, as follows:

- Section A (40%): 3 or 4 compulsory questions, each carrying varying marks, and involving authentic problem solving. The mathematical knowledge required in this section is restricted to the Pure Mathematics syllabus (Sections 1 to 16).

- Section B (60%): 7 questions based on the Mathematics 9233 syllabus. There will be 3 questions from the Pure Mathematics Section, 2 questions from the Particle Mechanics Section, and 2 questions from the Probability and Statistics Section. Each question is worth 15 marks, and candidates need to answer any four questions.

**9233 MATHEMATICS A LEVEL**

## Detailed Syllabus

Knowledge of the content of the Syndicate's Ordinary level Syllabus (or an equivalent syllabus) is assumed. Ordinary level material that is not repeated in the syllabus below will not be tested directly but it may be required indirectly in response to questions on other topics.

***PURE MATHEMATICS*** (Sections 1 to 16)
Section 1 to 16 will be assessed in Paper 1 as well as in Section A of Paper 2.

| THEME OR TOPIC | CURRICULUM OBJECTIVES |
|---|---|
| | *Candidates should be able to :* |
| 1 Functions and graphs | – understand the terms function, domain, range and one-one function;<br>– find composite functions and inverses of functions, including conditions for their existence;<br>– understand and use the relation $(\mathrm{fg})^{-1} = \mathrm{g}^{-1}\mathrm{f}^{-1}$ where appropriate;<br>– illustrate in graphical terms the relation between a one-one function and its inverse;<br>– understand the relationship between a graph and an associated algebraic equation, and in particular show familiarity with the forms of the graphs of<br>$y = kx^n$, where $n$ is a positive or negative integer or a simple rational number,<br>$ax + by = c$,<br>$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (knowledge of geometrical properties of conics is not required);<br>– understand and use the relationships between the graphs of $y = \mathrm{f}(x)$, $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = \mathrm{f}(ax)$, where $a$ is a constant, and express the transformations involved in terms of translations, reflections and scalings;<br>– relate the equation of a graph to its symmetries;<br>– understand, and use in simple cases, the expression of the coordinates of a point on a curve in terms of a parameter. |
| 2 Partial fractions | – recall an appropriate form for expressing rational functions in partial fractions, and carry out the decomposition, in cases where the denominator is no more complicated than<br>$(ax + b)(cx + d)(ex + f)$,<br>$(ax + b)(cx + d)^2$,<br>$(ax + b)(x^2 + c^2)$,<br>including cases where the degree of the numerator exceeds that of the denominator. |

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| 3 | Inequalities; the modulus function | – use properties of inequalities, and in particular understand that $x > y$ and $z > 0$ imply that $xz > yz$ while $x > y$ and $z < 0$ imply $xz < yz$;<br>– find the solution set of inequalities that are reducible to the form $f(x) > 0$, where $f(x)$ can be factorised, and illustrate such solutions graphically;<br>– understand the meaning of $\|x\|$ and sketch the graph of functions of the form $y = \|ax + b\|$ ;<br>– use relations such as $\|x - a\| < b \Leftrightarrow a - b < x < a + b$ and $\|a\| = \|b\| \Leftrightarrow a^2 = b^2$ in the course of solving equations and inequalities. |
| 4 | Logarithmic and exponential functions | – recall and use the laws of logarithms (including change of base) and sketch graphs of simple logarithmic and exponential functions;<br>– recall and use the definition $a^x = e^{x \ln a}$;<br>– use logarithms to solve equations reducible to the form $a^x = b$, and similar inequalities. |
| 5 | Sequences and series | – understand the idea of a sequence of terms, and use notations such as $u_n$ to denote the $n$th term of a sequence;<br>– recognise arithmetic and geometric progressions;<br>– use formulae for the $n$th term and for the sum of the first $n$ terms to solve problems involving arithmetic or geometric progressions;<br>– recall the condition for convergence of a geometric series, and use the formula for the sum to infinity of a convergent geometric series;<br>– use $\Sigma$ notation;<br>– use the binomial theorem to expand $(a + b)^n$, where $n$ is a positive integer;<br>– use the binomial theorem to expand $(1 + x)^n$, where $n$ is rational, and recall the condition $\|x\| < 1$ for the validity of this expansion;<br>– recognise and use the notations $n!$ (with $0! = 1$) and $\binom{n}{r}$. |
| 6 | Permutations and Combinations | – understand the terms ‘permutation’ and ‘combination’;<br>– solve problems involving arrangements (of objects in a line or in a circle), including those involving<br>repetition (e.g. the number of ways of arranging the letters of the word NEEDLESS),<br>restriction (e.g. the number of ways several people can stand in a line if 2 particular people must – or must not – stand next to each other). |
| 7 | Trigonometry | – use the sine and cosine formulae;<br>– calculate the angle between a line and a plane, the angle between two planes, and the angle between two skew lines in simple cases. |

## 9233 MATHEMATICS A LEVEL

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| 8 | Trigonometrical functions | – understand the definition of the six trigonometrical functions for angles of any magnitude;<br>– recall and use the exact values of trigonometrical functions of 30°, 45° and 60°, e.g. $\cos 30° = \frac{1}{2}\sqrt{3}$;<br>– use the notations $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ to denote the principal values of the inverse trigonometrical relations;<br>– relate the periodicity and symmetries of the sine, cosine and tangent functions to the form of their graphs, and use the concepts of periodicity and/or symmetry in relation to these functions and their inverses;<br>– use trigonometrical identities for the simplification and exact evaluation of expressions, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of<br>$\frac{\sin\theta}{\cos\theta} = \tan\theta$ and $\frac{\cos\theta}{\sin\theta} = \cot\theta$,<br>$\sin^2\theta + \cos^2\theta = 1$ and equivalent statements,<br>the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$,<br>the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$,<br>the formulae for $\sin A \pm \sin B$ and $\cos A \pm \cos B$,<br>the expression of $a\cos\theta + b\sin\theta$ in the forms $R\cos(\theta \pm \alpha)$ and $R\sin(\theta \pm \alpha)$;<br>– find the general solution of simple trigonometrical equations, including graphical interpretation;<br>– use the small-angle approximations $\sin x \approx x$, $\cos x \approx 1 - \frac{1}{2}x^2$, $\tan x \approx x$. |
| 9 | Differentiation | – understand the idea of a limit and the derivative defined as a limit, including geometrical interpretation in terms of the gradient of a curve at a point as the limit of the gradient of a suitable sequence of chords;<br>– use the standard notations $f'(x)$, $f''(x)$ etc., and $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ etc., for derived functions;<br>– use the derivatives of $x^n$ (for any rational $n$), $\sin x$, $\cos x$, $\tan x$, $e^x$, $a^x$, $\ln x$, $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$; together with constant multiples, sums, differences, products, quotients and composites;<br>– find and use the first derivative of a function which is defined implicitly or parametrically;<br>– locate stationary points, and distinguish between maxima, minima and stationary points of inflexion (knowledge of conditions for general points of inflexion is not required);<br>– find equations of tangents and normals to curves, and use information about gradients for sketching graphs; |

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| | – solve problems involving maxima and minima, connected rates of change, small increments and approximations;<br>– derive and use the first few terms of the Maclaurin series for a function. |
| 10 Integration | – understand indefinite integration as the reverse process of differentiation;<br>– integrate $x^n$ (including the case where $n = -1$), $e^x$, $\sin x$, $\cos x$, $\sec^2 x$, together with<br>sums, differences and constant multiples of these,<br>expressions involving a linear substitution (e.g. $e^{2x-1}$),<br>applications involving the use of partial fractions,<br>applications involving the use of trigonometrical identities (e.g. $\int \cos^2 x \, dx$);<br>– recognise an integrand of the form $\frac{k f'(x)}{f(x)}$ and integrate, e.g. $\frac{x}{x^2+1}$ or $\tan x$;<br>– integrate $\frac{1}{a^2+x^2}$ and $\frac{1}{\sqrt{a^2-x^2}}$;<br>– recognise when an integrand can usefully be regarded as a product, and use integration by parts to integrate, e.g., $x \sin 2x$, $x^2 e^x$, $\ln x$;<br>– use the method of integration by substitution to simplify and evaluate either a definite or an indefinite integral (including simple cases in which the candidates have to select the substitution themselves,<br>e.g., $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$);<br>– evaluate definite integrals (including e.g. $\int_0^1 x^{-\frac{1}{2}} dx$ and $\int_0^{\infty} e^{-x} dx$);<br>– understand the idea of the area under a curve as the limit of a sum of the areas of rectangles and use simple applications of this idea;<br>– use integration to find plane areas and volumes of revolution in simple cases;<br>– use the trapezium rule to estimate the values of definite integrals, and identify the sign of the error in simple cases by graphical considerations. |
| 11 Vectors | – use rectangular cartesian coordinates to locate points in three dimensions, and use standard notations for vectors, i.e.<br>$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, $\overrightarrow{AB}$, $\mathbf{a}$ ;<br>– carry out addition and subtraction of vectors and multiplication of a vector by a scalar, and interpret these operations in geometrical terms;<br>– use unit vectors, position vectors and displacement vectors; |

**9233 MATHEMATICS A LEVEL**

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| | | – recall the definition of and calculate the magnitude of a vector and the scalar product of two vectors;<br>– use the scalar product to determine the angle between two directions and to solve problems concerning perpendicularity of vectors;<br>– understand the significance of all the symbols used when the equation of a straight line is expressed in either of the forms $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ and $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$, and convert equations of lines from vector to cartesian form and *vice versa*;<br>– solve simple problems involving finding and using either form of the equation of a line;<br>– use equations of lines to solve problems concerning distances, angles and intersections, and in particular<br>determine whether two lines are parallel, intersect or are skew, and find the point of intersection of two lines when it exists,<br>find the perpendicular distance from a point to a line,<br>find the angle between two lines;<br>– use the ratio theorem in geometrical applications. |
| 12 | Mathematical induction | – understand the steps needed to carry out a proof by the method of induction;<br>– use the method of mathematical induction to establish a given result e.g. the sum of a finite series, or the form of an *n*th derivative. |
| 13 | Complex numbers | – understand the idea of a complex number, recall the meaning of the terms ‘real part’, ‘imaginary part’, ‘modulus’, ‘argument’, ‘conjugate’, and use the fact that two complex numbers are equal if and only if both real and imaginary parts are equal;<br>– carry out operations of addition, subtraction, multiplication and division of two complex numbers expressed in cartesian form $(x + \mathrm{i}y)$;<br>– recall and use the relation $zz^* = \|z\|^2$;<br>– use the result that, for a polynomial equation with real coefficients, any non-real roots occur in conjugate pairs;<br>– represent complex numbers geometrically by means of an Argand diagram;<br>– carry out operations of multiplication and division of two complex numbers expressed in polar form $(r(\cos\theta + \mathrm{i}\sin\theta) = r\mathrm{e}^{\mathrm{i}\theta})$;<br>– understand in simple terms the geometrical effects of conjugating a complex number and of adding, subtracting, multiplying, dividing two complex numbers;<br>– illustrate simple equations and inequalities involving complex numbers by means of loci in an Argand diagram (e.g. $\|z - a\| < k$, $\|z - a\| = \|z - b\|$, $\arg(z - a) = \alpha$, but excluding $\arg(z - a) - \arg(z - b) = \gamma$). |

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| 14 | Curve sketching | – understand the relationships between the graphs of $y = f(x)$, $y^2 = f(x)$ and $y = \lvert f(x) \rvert$;<br>– determine, in simple cases, the equations of asymptotes parallel to the axes;<br>– use the equation of a curve, in simple cases, to make deductions concerning symmetry or concerning any restrictions on the possible values of $x$ and/or $y$ that there may be;<br>– sketch curves of the form $y = f(x)$, $y^2 = f(x)$ or $y = \lvert f(x) \rvert$ (detailed plotting of curves will not be required, but sketches will generally be expected to show significant features, such as turning points, asymptotes and intersections with the axes). |
| 15 | First order differential equations | – formulate a simple statement involving a rate of change as a differential equation, including the introduction if necessary of a constant of proportionality;<br>– find by integration a general form of solution for a first order differential equation in which the variables are separable;<br>– find the general solution of a first order linear differential equation by means of an integrating factor;<br>– reduce a given first order differential equation to one in which the variables are separable or to one which is linear by means of a given simple substitution;<br>– understand that the general solution of a differential equation is represented in graphical terms by a family of curves, and sketch typical members of a family in simple cases;<br>– use an initial condition to find a particular solution to a differential equation, and interpret a solution in terms of a problem modelled by a differential equation. |
| 16 | Numerical methods | – locate approximately a root of an equation by means of graphical considerations and/or searching for a sign change;<br>– use the method of linear interpolation to find an approximation to a root of an equation;<br>– understand the idea of, and use the notation for, a sequence of approximations which converges to the root of an equation;<br>– understand how a given simple iterative formula of the form $x_{n+1} = F(x_n)$ relates to the equation being solved, and use a given iteration to determine a root to a prescribed degree of accuracy (conditions for convergence are not included);<br>– understand, in geometrical terms, the working of the Newton-Raphson method, and derive and use iterations based on this method;<br>– appreciate that an iterative method may fail to converge to the required root. |

**9233 MATHEMATICS A LEVEL**

***PARTICLE MECHANICS*** (Sections 1 to 8)

Sections 1 to 4 will be assessed in Paper 2 Section B while Sections 1 to 8 will be assessed in Paper 2 Section C.

| | THEME OR TOPIC | CURRICULUM OBJECTIVES |
|---|---|---|
| | | *Candidates should be able to :* |
| 1 | Forces and equilibrium | – identify the forces acting in a given situation;<br>– understand the representation of forces by vectors, and find and use resultants and components;<br>– solve problems concerning the equilibrium of a particle under the action of coplanar forces (using equations obtained by resolving the forces, or by using a triangle or polygon of forces);<br>– recall that the contact force between two surfaces can be represented by two components (the ‘normal component’ and the ‘frictional component’) and use this representation in solving problems;<br>– use the model of a ‘smooth’ contact and understand the limitations of this model;<br>– understand the concept of limiting friction and limiting equilibrium, recall the definition of coefficient of friction; and use the relationship $F \leqslant \mu R$ or $F = \mu R$ as appropriate (knowledge of angle of friction will not be required);<br>– recall and use Newton’s third law. |
| 2 | Kinematics of motion in a straight line | – understand the concepts of distance and speed, as scalar quantities, and of displacement, velocity and acceleration, as vector quantities, and understand the relationships between them;<br>– sketch and interpret *x*-*t* and *v*-*t* graphs, and in particular understand and use the facts that<br>the area under a *v*-*t* graph represents displacement,<br>the gradient of an *x*-*t* graph represents velocity,<br>the gradient of a *v*-*t* graph represents acceleration;<br>– use appropriate formulae for motion with constant acceleration in a straight line. |
| 3 | Newton’s laws of motion | – recall and use Newton’s first and second laws of motion;<br>– apply Newton’s laws to the linear motion of a particle of constant mass moving under the action of constant forces (including friction);<br>– solve problems on the motion of two particles, connected by a light inextensible string which may pass over a fixed smooth light pulley or peg. |

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| 4 | Energy, work and power | – understand the concept of the work done by a force, and calculate the work done by a constant force when its point of application undergoes a displacement not necessarily parallel to the force (use of the scalar product is not required);<br>– understand the concepts of gravitational potential energy and kinetic energy, and recall and use appropriate formulae;<br>– understand and use the relationship between the change in energy of a system and the work done by the external forces, and use where appropriate the principle of conservation of energy;<br>– recall and use the definition of power as the rate at which a force does work, and use the relationship between power, force and velocity for a force acting in the direction of motion;<br>– solve problems involving, for example, the instantaneous acceleration of a car moving on a hill with resistance. |
| 5 | Linear motion under a variable force | – solve simple problems on the linear motion of a particle of constant mass moving under the action of variable forces by setting up and solving an appropriate differential equation (use of $\frac{\mathrm{d}x}{\mathrm{d}t}$ for velocity and $\frac{\mathrm{d}v}{\mathrm{d}t}$ or $v\frac{\mathrm{d}v}{\mathrm{d}x}$, as appropriate, for acceleration is expected, and any differential equations to be solved will be first order with separable variables). |
| 6 | Motion of a projectile | – model the motion of a projectile as a particle moving with constant acceleration, and understand the limitations of this model;<br>– use horizontal and vertical equations of motion to solve problems on the motion of projectiles (including finding the magnitude and direction of the velocity at a given time or position and finding the range on a horizontal plane);<br>– derive and use the cartesian equation of the trajectory of a projectile, including cases where the initial speed and/or angle of projection is unknown (knowledge of the range on an inclined plane is not required). |
| 7 | Hooke’s law | – recall and use Hooke’s law as a model relating the force in an elastic string or spring to the extension or compression, and understand and use the term ‘modulus of elasticity’;<br>– understand the concept of elastic potential energy, and recall and use the appropriate formula for its calculation;<br>– use considerations of work and energy to solve problems involving elastic strings and springs. |
| 8 | Uniform circular motion | – understand the concept of angular speed for a particle moving in a circle with constant speed, and recall and use the relation $v = r\omega$ (no proof required); |

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| | – understand that the acceleration of particle moving in a circle with constant speed is directed towards the centre of the circle and has magnitude $r\omega^2$ or $\frac{v^2}{r}$ (no proof required);<br>– use Newton's second law to solve problems which can be modelled as the motion of a particle moving in a circle with constant speed. |

***PROBABILITY AND STATISTICS*** (Sections 1 to 7)

Sections 1 to 4 will be assessed in Paper 2 Section B while Sections 1 to 7 will be assessed in Paper 2 Section D.

| THEME OR TOPIC | CURRICULUM OBJECTIVES |
|---|---|
| | *Candidates should be able to :* |
| 1 Probability | – use addition and multiplication of probabilities, as appropriate, in simple cases, and understand the representation of events by means of tree diagrams;<br>– understand the meaning of mutually exclusive and independent events, and calculate and use conditional probabilities in simple cases;<br>– understand and use the notations P($A$), P($A \cup B$), P($A \cap B$), P($A \mid B$) and the equations P($A \cup B$) = P($A$) + P($B$) – P($A \cap B$) and P($A \cap B$) = P($A$) P($B \mid A$) = P($B$) P($A \mid B$) (the general form of Bayes' theorem is not required). |
| 2 Discrete random variables | – understand the concept of a discrete random variable;<br>– construct a probability distribution table relating to a given situation, and calculate E($X$) and Var($X$);<br>– appreciate conditions under which a uniform distribution or a binomial distribution B($n,p$) may be a suitable probability model, and recall and use formulae for the calculation of binomial probabilities;<br>– understand conditions under which a Poisson distribution Po($\mu$) may be a suitable probability model, and recall and use the formula for the calculation of Poisson probabilities;<br>– recall and use the means and variances of binomial and Poisson distributions;<br>– use a Poisson distribution as an approximation to a binomial distribution, where appropriate (candidates should know that the conditions $n > 50$ and $np < 5$, approximately, can generally be taken to be suitable). |

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| 3 | The normal distribution | – recall the general shape of a normal curve, and understand how the shape and location of the distribution $N(\mu, \sigma^2)$ are affected by the values of $\mu$, $\sigma$ (in general terms only; no knowledge of mathematical properties of the normal density function is included);<br>– standardise a normal variable and use normal distribution tables;<br>– use the normal distribution as a probability model, where appropriate, and solve problems concerning a variable $X$, where $X \sim N(\mu, \sigma^2)$, including<br>finding the value of $P(X < x_1)$ given the values of $x_1$, $\mu$, $\sigma$,<br>use of the symmetry of the normal distribution,<br>finding a relationship between $x_1$, $\mu$, $\sigma$, given the value of $P(X < x_1)$,<br>repeated application of the above;<br>– recall conditions under which a normal distribution may be used to approximate a binomial distribution ($n$ sufficiently large to ensure that $np > 5$ and $nq > 5$, approximately) or Poisson distribution ($\mu$ >10, approximately), and calculate such approximations, including the use of a continuity correction. |
| 4 | Samples | – understand the distinction between a sample and a population, and appreciate the necessity for randomness in choosing samples;<br>– explain in simple terms why a given sampling method may be unsatisfactory (a detailed knowledge of sampling and survey methods is not required);<br>– recognise that the sample mean can be regarded as a random variable, and use the facts that $E(\overline{X}) = \mu$ and $\mathrm{Var}(\overline{X}) = \dfrac{\sigma^2}{n}$;<br>– use the fact that $\overline{X}$ is normally distributed if $X$ is normally distributed;<br>– use the Central Limit Theorem (without proof) to treat $\overline{X}$ as being normally distributed when the sample size is sufficiently large ('large' samples will usually be of size at least 50, but candidates should know that using the approximation of normality can sometimes be useful with samples that are smaller than this);<br>– calculate unbiased estimates of the population mean and population variance from a sample (only a simple understanding of the term 'unbiased' is required);<br>– determine, from a sample from a normal distribution of known variance or from a large sample, a confidence interval for the population mean;<br>– determine, from a large sample, a confidence interval for a population proportion. |

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| 5 | Linear combinations of random variables | – recall and use the results in the course of solving problems that, for either discrete or continuous random variables,<br>$E(aX + b) = aE(X) + b$ and $Var(aX + b) = a^2 Var(X)$,<br>$E(aX + bY) = aE(X) + bE(Y)$,<br>$Var(aX + bY) = a^2 Var(X) + b^2 Var(Y)$ for independent $X$ and $Y$;<br>– recall and use the results that<br>if $X$ has a normal distribution then so does $aX + b$,<br>if $X$ and $Y$ have independent normal distributions then $aX + bY$ has a normal distribution.<br>if $X$ and $Y$ have independent Poisson distributions then $X + Y$ has a Poisson distribution. |
| 6 | Continuous random variables | – understand and use the concept of a probability density function, and recall and use the properties of a density function (which may be defined 'piecewise');<br>– use a given probability density function to calculate the mean, mode and variance of a distribution, and in general use the result $E(g(X)) = \int_{-\infty}^{\infty} g(x)\, f(x) dx$ in simple cases, where $f(x)$ is the probability density function of $X$ and $g(X)$ is a function of $X$;<br>– understand and use the relationship between the probability density function and the distribution function and use either to evaluate the median, quartiles and other percentiles;<br>– use a probability density function or a distribution function in the context of a model, including in particular the continuous uniform (rectangular) distribution. |
| 7 | Hypothesis testing | – understand and use the concepts of hypothesis (null and alternative), test statistic, significance level, and hypothesis test (1-tail and 2-tail);<br>– formulate hypotheses and apply a hypothesis test concerning the population mean using<br>a sample drawn from a normal distribution of known variance,<br>a large sample drawn from any distribution of unknown variance;<br>– formulate hypotheses concerning a population proportion, and apply a hypothesis test using a normal approximation to a binomial distribution. |

## MATHEMATICAL NOTATION

The list which follows summarizes the notation used in the Syndicate's Mathematics examinations. Although primarily directed towards Advanced level, the list also applies, where relevant, to examinations at all other levels, i.e. O level, AO level and N level.

# Mathematical Notation

1. *Set Notation*

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| $\in$ | is an element of |
| $\notin$ | is not an element of |
| $\{x_1, x_2, \ldots\}$ | the set with elements $x_1, x_2, \ldots$ |
| $\{x: \ldots\}$ | the set of all $x$ such that . . . |
| $n(A)$ | the number of elements in set $A$ |
| $\varnothing$ | the empty set |
| $\mathscr{E}$ | universal set |
| $A'$ | the complement of the set $A$ |
| $\mathbb{N}$ | the set of positive integers, $\{1, 2, 3, \ldots\}$ |
| $\mathbb{Z}$ | the set of integers $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$ |
| $\mathbb{Z}^+$ | the set of positive integers $\{1, 2, 3, \ldots\}$ |
| $\mathbb{Z}_n$ | the set of integers modulo $n$, $\{0, 1, 2, \ldots, n-1\}$ |
| $\mathbb{Q}$ | the set of rational numbers |
| $\mathbb{Q}^+$ | the set of positive rational numbers, $\{x \in \mathbb{Q}: x > 0\}$ |
| $\mathbb{Q}_0^+$ | the set of positive rational numbers and zero, $\{x \in \mathbb{Q}: x \geqslant 0\}$ |
| $\mathbb{R}$ | the set of real numbers |
| $\mathbb{R}^+$ | the set of positive real numbers $\{x \in \mathbb{R}: x > 0\}$ |
| $\mathbb{R}_0^+$ | the set of positive real numbers and zero $\{x \in \mathbb{R}: x \geqslant 0\}$ |
| $\mathbb{R}^n$ | the real $n$ tuples |
| $\mathbb{C}$ | the set of complex numbers |
| $\subseteq$ | is a subset of |
| $\subset$ | is a proper subset of |
| $\nsubseteq$ | is not a subset of |
| $\not\subset$ | is not a proper subset of |
| $\cup$ | union |
| $\cap$ | intersection |
| $[a, b]$ | the closed interval $\{x \in \mathbb{R}: a \leqslant x \leqslant b\}$ |
| $[a, b)$ | the interval $\{x \in \mathbb{R}: a \leqslant x < b\}$ |
| $(a, b]$ | the interval $\{x \in \mathbb{R}: a < x \leqslant b\}$ |
| $(a, b)$ | the open interval $\{x \in \mathbb{R}: a < x < b\}$ |
| $yRx$ | $y$ is related to $x$ by the relation $R$ |
| $y \sim x$ | $y$ is equivalent to $x$, in the context of some equivalence relation |

2. *Miscellaneous Symbols*

| | |
|---|---|
| $=$ | is equal to |
| $\neq$ | is not equal to |
| $\equiv$ | is identical to or is congruent to |
| $\approx$ | is approximately equal to |
| $\cong$ | is isomorphic to |
| $\propto$ | is proportional to |
| $<; \ll$ | is less than; is much less than |
| $\leqslant, \ngtr$ | is less than or equal to or is not greater than |
| $>; \gg$ | is greater than; is much greater than |
| $\geqslant, \nless$ | is greater than or equal to or is not less than |
| $\infty$ | infinity |

3. *Operations*

| | |
|---|---|
| $a + b$ | $a$ plus $b$ |
| $a - b$ | $a$ minus $b$ |
| $a \times b$, $ab$, $a.b$ | $a$ multiplied by $b$ |
| $a \div b$, $\frac{a}{b}$, $a/b$ | $a$ divided by $b$ |
| $a : b$ | the ratio of $a$ to $b$ |
| $\sum_{i=1}^{n} a_i$ | $a_1 + a_2 + \ldots + a_n$ |
| $\sqrt{a}$ | the positive square root of the real number $a$ |
| $\lvert a \rvert$ | the modulus of the real number $a$ |
| $n!$ | $n$ factorial for $n \in \mathbb{N}$ $(0! = 1)$ |
| $\binom{n}{r}$ | the binomial coefficient $\frac{n!}{r!\,(n-r)!}$, for $n, r \in \mathbb{N}$, $0 \leqslant r \leqslant n$<br>$\frac{n(n-1)\ldots(n-r+1)}{r!}$, for $n \in \mathbb{Q}$, $r \in \mathbb{N}$ |

4. *Functions*

| | |
|---|---|
| f | function f |
| f($x$) | the value of the function f at $x$ |
| f : $A \to B$ | f is a function under which each element of set $A$ has an image in set $B$ |
| f : $x \mapsto y$ | the function f maps the element $x$ to the element $y$ |
| $\mathrm{f}^{-1}$ | the inverse of the function f |
| g ∘ f , gf | the composite function of f and g which is defined by<br>$(\mathrm{g} \circ \mathrm{f})(x)$ or $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$ |
| $\lim_{x \to a} \mathrm{f}(x)$ | the limit of f($x$) as $x$ tends to $a$ |
| $\Delta x$; $\delta x$ | an increment of $x$ |
| $\frac{\mathrm{d}y}{\mathrm{d}x}$ | the derivative of $y$ with respect to $x$ |
| $\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$ | the $n$th derivative of $y$ with respect to $x$ |
| $\mathrm{f}'(x), \mathrm{f}''(x), \ldots, \mathrm{f}^{(n)}(x)$ | the first, second, . . . , $n$th derivatives of f($x$) with respect to $x$ |
| $\int y\,\mathrm{d}x$ | indefinite integral of $y$ with respect to $x$ |
| $\int_a^b y\,\mathrm{d}x$ | the definite integral of $y$ with respect to $x$ for values of $x$ between $a$ and $b$ |
| $\frac{\partial y}{\partial x}$ | the partial derivative of $y$ with respect to $x$ |
| $\dot{x}, \ddot{x}, \ldots$ | the first, second, . . . derivatives of $x$ with respect to time. |

5. *Exponential and Logarithmic Functions*

| | |
|---|---|
| e | base of natural logarithms |
| $\mathrm{e}^x$, exp $x$ | exponential function of $x$ |
| $\log_a x$ | logarithm to the base $a$ of $x$ |
| $\ln x$ | natural logarithm of $x$ |
| $\lg x$ | logarithm of $x$ to base 10 |

## MATHEMATICAL NOTATION

6. *Circular and Hyperbolic Functions and Relations*

| | |
|---|---|
| sin, cos, tan, cosec, sec, cot | the circular functions |
| $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$, $\operatorname{cosec}^{-1}$, $\sec^{-1}$, $\cot^{-1}$ | the inverse circular relations |
| sinh, cosh, tanh, cosech, sech, coth | the hyperbolic functions |
| $\sinh^{-1}$, $\cosh^{-1}$, $\tanh^{-1}$, $\operatorname{cosech}^{-1}$, $\operatorname{sech}^{-1}$, $\coth^{-1}$ | the inverse hyperbolic relations |

7. *Complex Numbers*

| | |
|---|---|
| i | square root of –1 |
| $z$ | a complex number, $z = x + \mathrm{i}y$ $= r(\cos\theta + \mathrm{i}\sin\theta),\ r \in \mathbb{R}_0^+$ $= r\mathrm{e}^{\mathrm{i}\theta},\ r \in \mathbb{R}_0^+$ |
| Re $z$ | the real part of $z$, $\mathrm{Re}(x + \mathrm{i}y) = x$ |
| Im $z$ | the imaginary part of $z$, $\mathrm{Im}(x + \mathrm{i}y) = y$ |
| $\lvert z \rvert$ | the modulus of $z$, $\lvert x + \mathrm{i}y \rvert = \sqrt{(x^2 + y^2)}$, $\lvert r(\cos\theta + \mathrm{i}\sin\theta) \rvert = r$ |
| arg $z$ | the argument of $z$, $\arg(r(\cos\theta + \mathrm{i}\sin\theta)) = \theta$, $-\pi < \theta \leqslant \pi$ |
| $z^*$ | the complex conjugate of $z$, $(x + \mathrm{i}y)^* = x - \mathrm{i}y$ |

8. *Matrices*

| | |
|---|---|
| $\mathbf{M}$ | a matrix $\mathbf{M}$ |
| $\mathbf{M}^{-1}$ | the inverse of the square matrix $\mathbf{M}$ |
| $\mathbf{M}^{\mathrm{T}}$ | the transpose of the matrix $\mathbf{M}$ |
| det $\mathbf{M}$ | the determinant of the square matrix $\mathbf{M}$ |

9. *Vectors*

| | |
|---|---|
| $\mathbf{a}$ | the vector $\mathbf{a}$ |
| $\overrightarrow{AB}$ | the vector represented in magnitude and direction by the directed line segment $AB$ |
| $\hat{\mathbf{a}}$ | a unit vector in the direction of the vector $\mathbf{a}$ |
| $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ | unit vectors in the directions of the cartesian coordinate axes |
| $\lvert \mathbf{a} \rvert$ | the magnitude of $\mathbf{a}$ |
| $\lvert \overrightarrow{AB} \rvert$ | the magnitude of $\overrightarrow{AB}$ |
| $\mathbf{a} . \mathbf{b}$ | the scalar product of $\mathbf{a}$ and $\mathbf{b}$ |
| $\mathbf{a} \times \mathbf{b}$ | the vector product of $\mathbf{a}$ and $\mathbf{b}$ |

10. *Probability and Statistics*

| | |
|---|---|
| $A$, $B$, $C$, etc. | events |
| $A \cup B$ | union of events $A$ and $B$ |
| $A \cap B$ | intersection of the events $A$ and $B$ |
| P($A$) | probability of the event $A$ |
| $A'$ | complement of the event $A$, the event ‘not $A$’ |
| P($A$\|$B$) | probability of the event $A$ given the event $B$ |
| $X$, $Y$, $R$, etc. | random variables |
| $x$, $y$, $r$, etc. | values of the random variables $X$, $Y$, $R$, etc. |
| $x_1, x_2, \ldots$ | observations |
| $f_1, f_2, \ldots$ | frequencies with which the observations $x_1, x_2, \ldots$ occur |
| p($x$) | the value of the probability function $\mathrm{P}(X = x)$ of the discrete random variable $X$ |

## MATHEMATICAL NOTATION

| | |
|---|---|
| $p_1, p_2, \ldots$ | probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable $X$. |
| $\mathrm{f}(x), \mathrm{g}(x), \ldots$ | the value of the probability density function of the continuous random variable $X$ |
| $\mathrm{F}(x), \mathrm{G}(x), \ldots$ | the value of the (cumulative) distribution function $\mathrm{P}(X \leqslant x)$ of the random variable $X$ |
| $\mathrm{E}(X)$ | expectation of the random variable $X$ |
| $\mathrm{E}[\mathrm{g}(X)]$ | expectation of $\mathrm{g}(X)$ |
| $\mathrm{Var}(X)$ | variance of the random variable $X$ |
| $\mathrm{G}(t)$ | the value of the probability generating function for a random variable which takes integer values |
| $\mathrm{B}(n, p)$ | binomial distribution, parameters $n$ and $p$ |
| $\mathrm{N}(\mu, \sigma^2)$ | normal distribution, mean $\mu$ and variance $\sigma^2$ |
| $\mu$ | population mean |
| $\sigma^2$ | population variance |
| $\sigma$ | population standard deviation |
| $\overline{x}$ | sample mean |
| $s^2$ | unbiased estimate of population variance from a sample, $s^2 = \dfrac{1}{n-1}\sum(x - \overline{x})^2$ |
| $\phi$ | probability density function of the standardised normal variable with distribution N(0, 1) |
| $\Phi$ | corresponding cumulative distribution function |
| $\rho$ | linear product-moment correlation coefficient for a population |
| $r$ | linear product-moment correlation coefficient for a sample |
| $\mathrm{Cov}(X, Y)$ | covariance of $X$ and $Y$ |