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4C1 (2)

Weekend Assignment (Remainder & Factor Theorem) Date No.

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24. (i) $f(x) = 4x^4 + px^3 - 19x^2 + qx - 6$

$f(-2) = 0$

$4(-2)^4 + p(-2)^3 - 19(-2)^2 + q(-2) - 6 = 0$

$64 - 8p - 76 - 2q - 6 = 0$

$-8p - 2q = 18$

$8p + 2q = -18$

$4p + q = -9$ — (1)

$\frac{df(x)}{dx} = 16x^3 + 3px^2 - 38x + q$

When $\frac{df(x)}{dx} = 0$,

$16x^3 + 3px^2 - 38x + q = 0$

When $x = \frac{1}{2}$,

$16(\frac{1}{2})^3 + 3p(\frac{1}{2})^2 - 38(\frac{1}{2}) + q = 0$

$2 + \frac{3}{4}p - 19 + q = 0$

$\frac{3}{4}p + q = 17$

$3p + 4q = 68$ — (2)

from (1): $4p + q = -9$

$q = -4p - 9$ — (3)

Sub (3) into (2):

$3p + 4(-4p - 9) = 68$

$3p - 16p - 36 = 68$

$-13p = 104$

$p = -8$

When $p = -8$,

$q = -4(-8) - 9$

$= 23$

Date No.

(ii) $f(x) = 4x^4 - 8x^3 - 19x^2 + 23x - 6$

$f(3) = 4(3)^4 - 8(3)^3 - 19(3)^2 + 23(3) - 6$

$= 0$

$\therefore$ The remainder when $f(x)$ is divided by $(x-3)$ is 0.

25. $x^2 + x - 2 = (x+2)(x-1)$

Let $f(x) = 2x^3 + px^2 + qx - 2$

$f(-2) = 0$

$2(-2)^3 + p(-2)^2 + q(-2) - 2 = 0$

$-16 + 4p - 2q - 2 = 0$

$4p - 2q = 18$

$2p - q = 9$

$q = 2p - 9$ —— (1)

$f(1) = 0$

$2(1)^3 + p(1)^2 + q(1) - 2 = 0$

$2 + p + q - 2 = 0$

$q = -p$ —— (2)

Sub (2) into (1):

$-p = 2p - 9$

$-3p = -9$

$p = 3$

When $p = 3$,

$q = -3$

$\therefore f(x) = 2x^3 + 3x^2 - 3x - 2$

Date No.

$$\begin{array}{r} 2x^2+5x+2 \\ (x-1)\overline{)2x^3+3x^2-3x-2} \\ -(2x^3-2x^2) \\ \hline 5x^2-3x \\ -(5x^2-5x) \\ \hline (2x-2) \\ -(2x-2) \\ \hline 0 \end{array}$$

$\therefore\ (2x^2+5x+2)(x-1)=0$

$(2x+1)(x+2)(x-1)=0$

$\therefore$ The other factor is $(2x+1)$.

$2(y+1)^3+3(y+1)^2-3(y-1)=8$

$2(y+1)^3+3(y+1)^2-3y+3-8=0$

$2(y+1)^3+3(y+1)^2-3y-5=0$

$2(y+1)^3+3(y+1)^2-3(y+1)-2=0$

$[2(y+1)+1][(y+1)+2][(y+1)-1]=0$

$2(y+1)=-1$ or $y+1=-2$ or $y+1=1$

$y+1=-\frac{1}{2}$ $\quad y=-3$ $\quad y=0$

$y=-1\frac{1}{2}$

Date No.

31. $f(x) = x^3 + ax^2 + bx + c$

Divided by $x^2 + 5x + 6$, $R = 3x + 9$

Factor x

Dividend = Divisor × Quotient + remainder

$x^3 + ax^2 + bx + c = (x^2 + 5x + 6)Q(x) + 3x + 9$

$= (x+3)(x+2)Q(x) + 3x + 9$

Let $x = -2$

$(-2)^3 + a(-2)^2 + b(-2) + c = 0 + 3(-2) + 9$

$-8 + 4a - 2b + c = -6 + 9$

$4a - 2b + c = 11$ —— (1)

Let $x = -3$

$(-3)^3 + a(-3)^2 + b(-3) + c = 0 + 3(-3) + 9$

$-27 + 9a - 3b + c = -9 + 9$

$9a - 3b + c = 27$ —— (2)

$f(0) = 0$

$0^3 + a(0)^2 + b(0) + c = 0$

$c = 0$ —— (3)

Sub (3) into (1):

$4a - 2b = 11$ —— (4)

Sub ~~3~~ into (2):

$9a - 3b = 27$

$b = 3a - 9$ —— (5)

Sub (5) into (4):

$4a - 2(3a - 9) = 11$

$4a - 6a + 18 = 11$

$a = 3\frac{1}{2}$

when $a = \frac{7}{2}$, $b = 3(\frac{7}{2}) - 9$

$= 1\frac{1}{2}$

$\therefore a = 3\frac{1}{2}$, $b = 1\frac{1}{2}$, $c = 0$

32. (i) Let $f(x) = 9x^3 + 18x^2 + bx - 8$

$f(-a) = 0$

$9(-a)^3 + 18(-a)^2 + b(-a) - 8 = 0$

$-9a^3 + 18a^2 - ab - 8 = 0$ — (1)

$f(a) = 128$

$9(a)^3 + 18(a)^2 + b(a) - 8 = 128$

$9a^3 + 18a^2 + ab = 136$

$ab = -9a^3 - 18a^2 + 136$ — (2)

Sub (2) into (1):

$-9a^3 + 18a^2 - (-9a^3 - 18a^2 + 136) - 8 = 0$

$-9a^3 + 18a^2 + 9a^3 + 18a^2 - 136 - 8 = 0$

$36a^2 - 144 = 0$

$(6a)^2 - 12^2 = 0$

$(6a-12)(6a+12) = 0$

$\therefore 6a = 12$ $\underline{\text{or}}$ $6a = -12$

$a = 2$ $\qquad a = -2$ (n.a.)

When $a = 2$,

$2b = -9(2)^3 - 18(2)^2 + 136$

$2b = -8$

$b = -4$

$\therefore a = 2, b = -4$ (shown) //

Date No.

(ii) $\therefore\ 9x^3 + 18x^2 - 4x - 8 = 0$

Let $f(x) = 9x^3 + 18x^2 - 4x - 8$

$\therefore\ (x+2)$ is a factor of $f(x)$

$$
\begin{array}{r|l}
 & 9x^2 - 4 \\
\hline
(x+2) & 9x^3 + 18x^2 - 4x - 8 \\
 & -(9x^3 + 18x^2) \\
\hline
 & -4x \\
 & -(-4x - 8) \\
\hline
 & 0
\end{array}
$$

$\therefore\ 9x^3 + 18x^2 - 4x - 8 = 0$

$(9x^2 - 4)(x+2) = 0$

$(3x-2)(3x+2)(x+2) = 0$

$\therefore\ x = \frac{2}{3}, -\frac{2}{3}$ or -2 //