

Chpt. 9 (Functions)

Date

No.

2 (i) $fg(x) = f(x+5)$

$$fg(x) = \sqrt{x+5}$$

$$\therefore fg: x \mapsto \sqrt{x+5}$$

(ii) Let $y = g(x)$

$$y = x+5$$

$$x = y-5$$

$$\therefore g^{-1}(x) = x-5$$

$$\therefore g^{-1}: x \mapsto x-5$$

(iii) $g^2(x) = g(x+5)$

$$g^2(x) = (x+5)+5$$

$$g^2(x) = x+10$$

$$\therefore g^2: x \mapsto x+10$$

(iv) $g^2 f(x) = g^2(\sqrt{x})$

$$g^2 f(x) = \sqrt{x} + 10$$

$$\therefore g^2 f: x \mapsto \sqrt{x} + 10$$

(v) Let $y = f(x)$

$$y = \sqrt{x}$$

$$x = y^2$$

$$\therefore f^{-1}(x) = x^2$$

$$gf^{-1}(x) = g(x^2)$$

$$gf^{-1}(x) = x^2 + 5$$

$$\therefore gf^{-1}: x \mapsto x^2 + 5$$

11. a. $h(x) = x^2 + 1$

$$h(0) = 1$$

$$h(2) = 5$$

Let $y = h(x)$

$$y = x^2 + 1$$

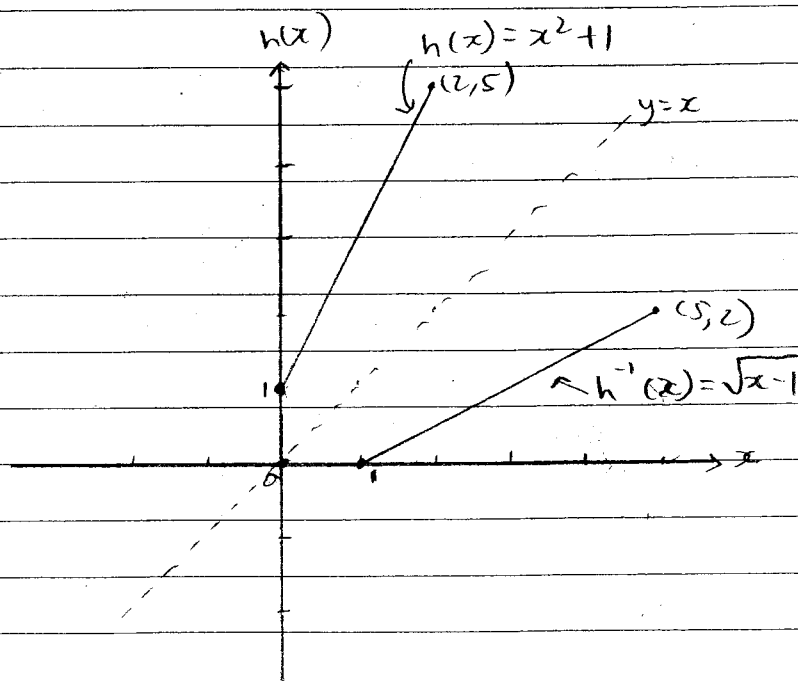
$$x^2 = y - 1$$

$$x = \sqrt{y-1}$$

$$\therefore h^{-1}(x) = \sqrt{x-1}$$

$$h^{-1}(1) = 0$$

$$h^{-1}(5) = 2$$



b. Let $y = x^2 - 3x + 2$
 $= x^2 - 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 2$
 $= \left(x - \frac{3}{2}\right)^2 - \frac{1}{4}$
 min. point is $\left(\frac{3}{2}, -\frac{1}{4}\right)$

When $x=0$,

$$y = 2$$

When $x=3$,

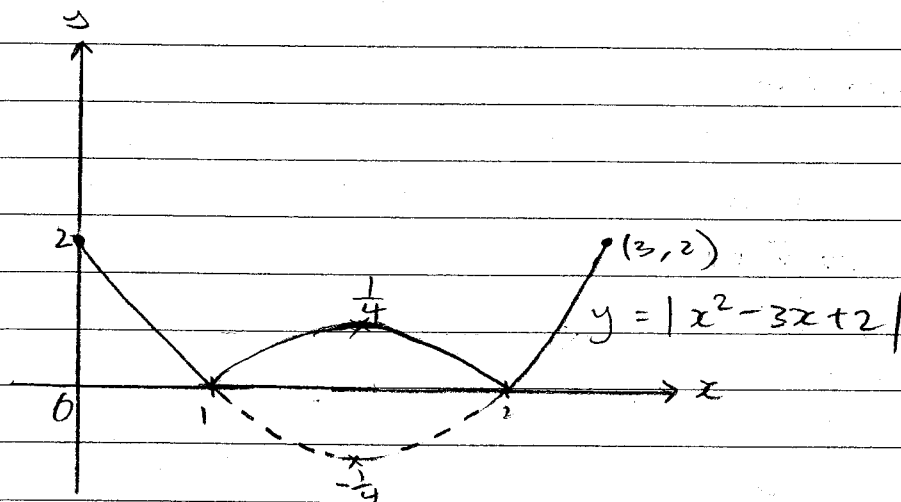
$$y = 2$$

When $y=0$,

$$\left(x - \frac{3}{2}\right)^2 = \frac{1}{4}$$

$$x - \frac{3}{2} = \pm \sqrt{\frac{1}{4}}$$

$$x = 2 \quad \text{or} \quad x = 1$$



Corresponding range of y : $0 \leq y \leq 2$

20. a.i) $fg(x) = f\left(\frac{2}{x}\right)$
 $= 3\left(\frac{2}{x}\right) - 1$
 $= \frac{6}{x} - 1$

$$\therefore fg: x \mapsto \frac{6}{x} - 1, \quad x \neq 0$$

$$gf(x) = g(3x-1)$$

$$= \frac{2}{3x-1}, \quad x \neq \frac{1}{3}$$

(ii)

$$fg(x) = gf(x)$$

$$\frac{6}{x} - 1 = \frac{2}{3x-1}$$

$$\frac{6-x}{x} = \frac{2}{3x-1}$$

$$18x - 6 - 3x^2 + x = 2x$$

$$-3x^2 + 19x - 6 = 2x$$

$$3x^2 - 17x + 6 = 0$$

$$x = \frac{17 \pm \sqrt{(-17)^2 - 4(3)(6)}}{2(3)}$$

$$x = \frac{17 \pm \sqrt{217}}{6}$$

$$x = \frac{17 + \sqrt{217}}{6}$$

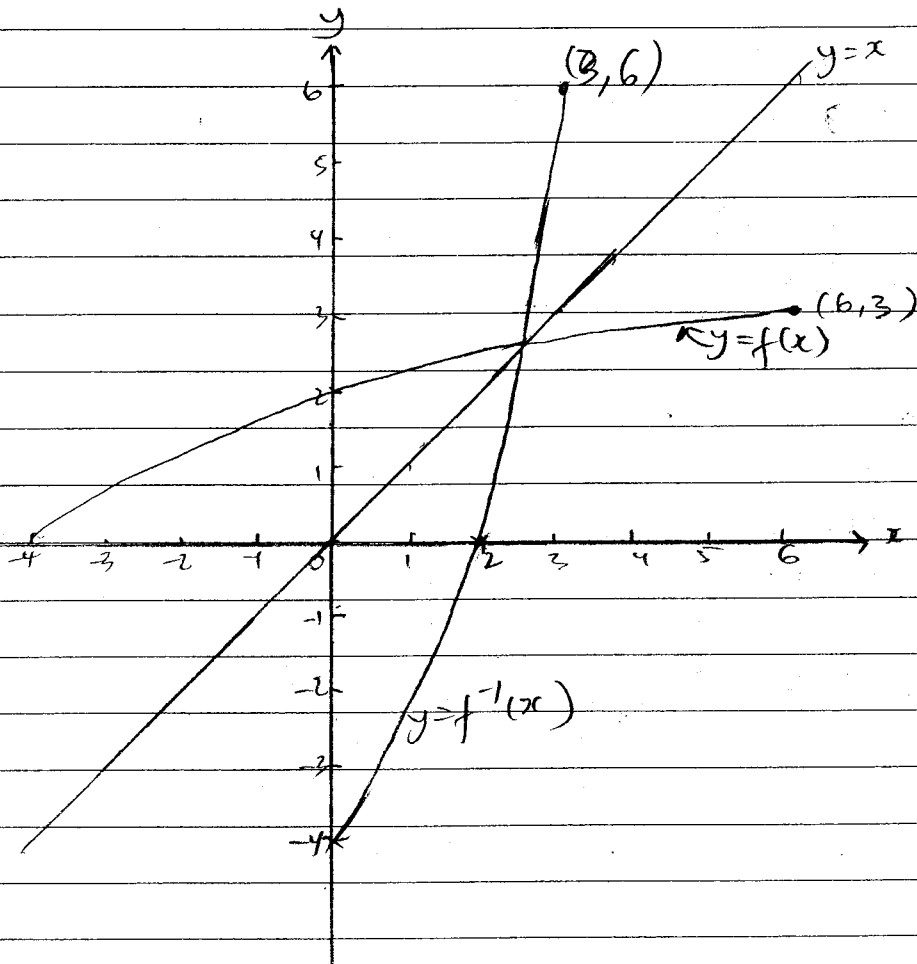
or

$$x = \frac{17 - \sqrt{217}}{6}$$

$$x = 5.29 \text{ (2dp)}$$

$$x = 0.38 \text{ (2dp)}$$

34. a.



$$-4 \leq f^{-1}(x) \leq 6$$

$$\begin{aligned}
 \text{b. (i)} \quad fg(x) &= f\left(\frac{1}{x-1}\right) \\
 &= 2\left(\frac{1}{x-1}\right) - 1 \\
 &= \frac{2}{x-1} - 1 \\
 &= \frac{2-x+1}{x-1} \\
 &= \frac{3-x}{x-1}, \quad x \neq 1
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad \text{Let } g(x) &= y \\
 y &= \frac{1}{x-1} \\
 xy - y &= 1 \\
 xy &= 1 + y \\
 x &= \frac{1+y}{y} \\
 \therefore g^{-1}(x) &= \frac{1+x}{x}, \quad x \neq 0
 \end{aligned}$$

$$\begin{aligned}
 \text{c. (i)} \quad g^2(x) &= (x-3)-3 \\
 g^2(x) &= x-6 \quad \#
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad g^2f(x) &= g^2(x^2) \\
 g^2f(x) &= x^2 - 6 \quad \#
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad \text{Let } g(x) &= y \\
 y &= x-3 \\
 x &= y+3 \\
 \therefore g^{-1}(x) &= x+3
 \end{aligned}$$

$$fg^{-1}(x) = (x+3)^2 \quad \#$$

20b.

