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4C1 (2)

Misc. Ex 17.

Date No.

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E CHUA

2. $y = 4x^2 + 27x^{-1}$

$\frac{dy}{dx} = 8x - 27x^{-2}$

When $\frac{dy}{dx} = 0$,

$8x - \frac{27}{x^2} = 0$

$8x^3 - 27 = 0$

$8x^3 = 27$

$x^3 = \frac{27}{8}$

$x = \frac{3}{2}$

When $x = \frac{3}{2}$,

$y = 4\left(\frac{3}{2}\right)^2 + 27\left(\frac{3}{2}\right)^{-1}$

$= 27$

$\therefore$ Coordinates of the turning point is $\left(\frac{3}{2}, 27\right)$ //

$\frac{d^2y}{dx^2} = 8 + 54x^{-3}$

When $x = \frac{3}{2}$,

$\frac{d^2y}{dx^2} = 8 + 54\left(\frac{3}{2}\right)^{-3}$

$= 24$

> 0

$\therefore \left(\frac{3}{2}, 27\right)$ is a min. point //

$\therefore$ Minimum value of y is 27 //

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4\. a. $y = ax^3 + 2x^2 + a^2x + b$

$\frac{dy}{dx} = 3ax^2 + 4x + a^2$

When $\frac{dy}{dx} = 0$,

$3ax^2 + 4x + a^2 = 0$

When $x = -1$,

$3a(-1)^2 + 4(-1) + a^2 = 0$

$3a - 4 + a^2 = 0$

$a^2 + 3a - 4 = 0$

$(a-1)(a+4) = 0$

$a = 1$ or $a = -4$

$\frac{d^2y}{dx^2} = 6ax + 4$

When $x = -1$,

$\frac{d^2y}{dx^2} = -6a + 4$

When $a = 1$,

$\frac{d^2y}{dx^2} = -6 + 4$

$= -2$

< 0

When $a = 1$, the turning point is a max point.

$\therefore a = 1$ (n.a.)

When $a = -4$

$\frac{d^2y}{dx^2} = -6(-4) + 4$

$= 28$

> 0

$\therefore$ When $a = -4$, turning point is a min. point.

When $x = -1$, $y = 0$,

$a(-1)^3 + 2(-1)^2 + a^2(-1) + b = 0$

$-a + 2 - a^2 + b = 0$

When $a = -4$,

$-(-4) + 2 - (-4)^2 + b = 0$

$6 - 16 + b = 0$

$b = 10$

$\therefore a = -4,\ b = 10$

b. $y = -4x^3 + 2x^2 + 16x + 10$

$\frac{dy}{dx} = -12x^2 + 4x + 16$

When $\frac{dy}{dx} = 0$

$-12x^2 + 4x + 16 = 0$

$3x^2 - x - 4 = 0$

$(3x - 4)(x + 1) = 0$

$3x = 4$ or $x = -1$ (n.a.)

$x = \frac{4}{3}$

When $x = \frac{4}{3}$,

$y = -4\left(\frac{4}{3}\right)^3 + 2\left(\frac{4}{3}\right)^2 + 16\left(\frac{4}{3}\right) + 10$

$= 25\frac{11}{27}$

$\therefore$ Coordinates of the other turning point is $\left(1\frac{1}{3},\ 25\frac{11}{27}\right)$

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7. $100 = 4x + 4y$

$x + y = 25$

$y = 25 - x$ //

$A = x^2 + y^2$

$= x^2 + (25 - x)^2$ // (shown).

$A = x^2 + 625 - 50x + x^2$

$= 2x^2 - 50x + 625$

$\frac{dA}{dx} = 4x - 50$

When $\frac{dA}{dx} = 0$,

$4x = 50$

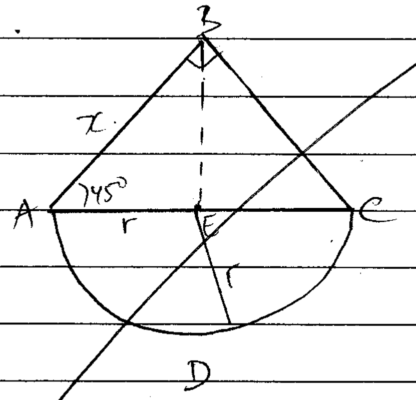
$x = 12.5$ //

$\frac{d^2A}{dx^2} = 4$

> 0

$\therefore$ When $x = 12.5$, area is minimum //

~~10.~~



$\cos 45° = \frac{r}{x}$

$x = \frac{r}{\cos 45°}$

$x = r\sqrt{2}$

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10. $80 = 2x + \frac{1}{2}\pi 2r$

$80 = 2x + \pi r$

$2x = 80 - \pi r$

$x = \frac{80 - \pi r}{2}$

$\text{Area} = \frac{1}{2}x^2 + \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi r^2 + \frac{1}{2}\left(\frac{80-\pi r}{2}\right)^2$

$= \frac{1}{2}\pi r^2 + \frac{(80-\pi r)^2}{8}$

$= \frac{1}{2}\pi r^2 + \frac{1}{8}(80-\pi r)^2$ // (shown)

$\frac{dA}{dr} = \pi r + \frac{1}{4}(80 - \pi r)(-\pi)$

$= \pi r - 20\pi - \frac{\pi^2 r}{4}$

When $\frac{dA}{dr} = 0$,

$\pi r + \frac{\pi^2 r}{4} - 20\pi = 0$

$4\pi r + \pi^2 r - 80\pi = 0$

$4r + \pi r = 80$

$(4+\pi)r = 80$

$r = \frac{80}{4+\pi}$

$= 11.202$ (5 sf)

$= 11.2$ (3 sf) //

$\frac{d^2A}{dr^2} = \pi - \frac{\pi^2}{4}$

$= 0.674$

> 0

$\therefore$ When $r = 11.2$, Area is minimum //

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11. Area $= \frac{1}{2}r^2\theta$

$L = r\theta + 2r$

$L = r(\theta+2)$

$\theta+2 = \frac{L}{r}$

$\theta = \frac{L}{r} - 2$

$A = \frac{1}{2}r^2\left(\frac{L}{r} - 2\right)$

$= \frac{Lr}{2} - r^2$

$= \frac{1}{2}rL - r^2$ // (shown)

a. $A = \frac{1}{2}Lr - r^2$

$\frac{dA}{dr} = \frac{1}{2}L - 2r$

When $\frac{dA}{dr} = 0$,

$\frac{1}{2}L - 2r = 0$

$L - 4r = 0$

$L = 4r$ //

$\frac{L}{r} = 4$

$\theta = 4 - 2$

$= 2$ rad //

$\frac{d^2A}{dr^2} = -2$

< 0

$\therefore$ When $L = 4r$, Area is maximum //

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b. Area of Δ OPQ $= \frac{1}{2} \times r^2 \times \sin\theta$

Area of sector OPQ $= \frac{1}{2} r^2 \theta$

Area of Δ OPQ $= \frac{\frac{1}{2} r^2 \sin\theta}{\frac{1}{2} r^2 \theta} \times 100\%$

$= \frac{\sin\theta}{\theta} \times 100\%$

$= \frac{\sin 2}{2} \times 100\%$

$= 45.465\%$ (5 sf)

$= 45.5\%$ (3 sf) //

12. $V = \pi r^2 h$

$250\cancel{\pi} = \cancel{\pi} r^2 h$

$h = \frac{250}{r^2}$.

$A = 2\pi r^2 + 2\pi r h$

$= 2\pi r^2 + 2\pi r \left(\frac{250}{r^2}\right)$

$= 2\pi r^2 + \frac{500\pi}{r}$ // (shown)

a. $\frac{dA}{dr} = 4\pi r - 500\pi r^{-2}$

When $\frac{dA}{dr} = 0$,

$4\pi r - \frac{500\pi}{r^2} = 0$

$4\pi r^3 - 500\pi = 0$

$4\pi r^3 = 500\pi$

$r^3 = 125$

$r = 5$

$\frac{d^2A}{dr^2} = 4\pi + 1000\pi r^{-3}$

When $r = 5$,

$\frac{d^2A}{dr^2} = 4\pi + \frac{1000\pi}{(5)^3}$

$= 37.7$

> 0

$\therefore$ When $r = 5$, Area is minimum.

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When $r=5$,
$h=\frac{250}{25}$
$=10$

$\therefore r=5$ cm, $h=10$ cm //

6. Cost $= 0.03 \times 2\pi r^2 + 0.05 \times \frac{500\pi}{r}$
$C = 0.06\pi r^2 + \frac{25\pi}{r}$

$\frac{dC}{dr} = 0.12\pi r - 25\pi r^{-2}$
When $\frac{dC}{dr}=0$,
$0.12\pi r - \frac{25\pi}{r^2} = 0$
$0.12\pi r^3 - 25\pi = 0$
$0.12\pi r^3 = 25\pi$
$r^3 = \frac{25}{0.12}$
$r = \sqrt[3]{\frac{25}{0.12}}$
$r = 5.9282$ (5 sf)
$= 5.93$ (3 sf)

$\frac{d^2C}{dr^2} = 0.12\pi + 50\pi r^{-3}$
When $r = 5.9282$
$\frac{d^2A}{dr^2} = 0.12\pi + \frac{50\pi}{(5.9282)^3}$
$= 4.8466$ (5 sf)
> 0
$\therefore$ Cost is minimum when $r = 5.9282$

When $r = 5.9282$
$h = \frac{250}{(5.9282)^2}$
$= 7.1137$ (5 sf)
$= 7.11$ (3 sf) //

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18. Total $V = (2x)(2x)(x) + \pi x^2 y$

$27 = 4x^3 + \pi x^2 y$

$27 = x^2(4x + \pi y)$

$4x + \pi y = \frac{27}{x^2}$

$\pi y = \frac{27}{x^2} - 4x$

$y = \frac{27}{\pi x^2} - \frac{4x}{\pi}$

$A = (2x)(2x) + (4)(2x)(x) + \pi x^2 + 2\pi x\left(\frac{27}{\pi x^2} - \frac{4x}{\pi}\right) + (4x^2 - \pi x^2)$

$= 4x^2 + 8x^2 + \pi x^2 + \frac{54}{x} - 8x^2 + 4x^2 - \pi x^2$

$= 8x^2 + \frac{54}{x}$

$= \frac{54}{x} + 8x^2$ // (shown)

a. $\frac{dA}{dx} = -54x^{-2} + 16x$

When $\frac{dA}{dx} = 0$,

$-\frac{54}{x^2} + 16x = 0$

$16x^3 - 54 = 0$

$16x^3 = 54$

$x^3 = 3.375$

$x = 1.5$ //

b. $A = \frac{54}{1.5} + 8(1.5)^2$

$= 54$

$y = \frac{27}{\pi(1.5)^2} - \frac{4(1.5)}{\pi}$

$= \frac{27}{2.25\pi} - \frac{6}{\pi}$

$= \frac{27}{2.25\pi} - \frac{13.5}{2.25\pi}$

$= \frac{13.5}{2.25\pi}$

$= \frac{6}{\pi}$

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$\frac{dA}{dx} = -54x^{-2} + 16x$

$\frac{d^2A}{dx^2} = 108x^{-3} + 16$

When $x = 1.5$

$\frac{d^2A}{dx^2} = 108(1.5)^{-3} + 16$

$= 48$

> 0

$\therefore$ When $x = 1.5$, Area is minimum //