

Topic 27: Inequalities

1a. $5x + 3 \leq \frac{4}{5}(3x + 23)$

$$25x + 15 \leq 12x + 92$$

$$13x \leq 77$$

$$x \leq \frac{77}{13} \Rightarrow x \leq 5\frac{12}{13}$$

\therefore The largest rational value of x is $5\frac{12}{13}$.

b. If x is a prime no., the greatest possible value of x is 5.

2a. $6n \leq 37$

$$n \leq 6\frac{1}{6}$$

\therefore The largest integer n is 6.

b. $3m - 1 > 35$

$$m > 12$$

\therefore The smallest integer m is 13.

3. $-4 \leq x \leq 1$

$$2 \leq y \leq 9$$

a. Least possible value of $x^2 - y = 0 - 9 = -9$

b. Least possible value of $y - x^2 = 2 - (-4)^2 = -14$

c. Greatest possible value of $\frac{x^2}{y} = \frac{(-4)^2}{2} = 8$

\therefore

4. $-8 \leq x \leq 4$

$$-6 \leq y \leq 6$$

a. Greatest possible value of $2x - y = 2(4) - (-6) = 14$

b. Greatest possible value of $2x^2 - y^2 = 2(-8)^2 - (0)^2 = 128$

c. Smallest possible value of $\frac{x}{x} + \frac{y}{y} = \frac{-8}{-8} + \frac{0}{0} = -2$

5. $2 \leq x \leq 8$

$$-4 \leq y \leq -2$$

a. Greatest possible value of $x - y = 8 - (-4) = 12$

b. Least possible value of $\frac{x}{y} = \frac{8}{-2} = -4$

c. Least possible value of $\frac{x^2 + y^2}{y - x} = \frac{8^2 + (-4)^2}{-2 - 2} = -20$

6. $3 \leq x \leq 8$

$-2 \leq y \leq 5$

a. Smallest possible value of $\frac{y}{x} = -\frac{2}{3}$

b. Largest possible value of $2x - y = 2(8) - (-2)$
 $= 18$

c. Smallest possible value of $(2x - y)^2 = (2(3) - 5)^2 = 1$

d. Largest possible value of $x^2 - y^2 = 8^2 - 0^2 = 64$

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